Sun and Eclipses

Activities and Models to explain eclipses

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Prologue

The National Plan for Science, Technology and Innovation "Innovative Argentina 2020" said, among other objectives, the need of efforts oriented to implement and coordinate mechanisms for distribution and social appropriation of scientific knowledge, one way to prioritize the perception of science among social actors, with the purpose of that Science and Technology are tools for inclusive innovation and provide answers to the needs of social development and improving the quality of life of the population.

For these reasons, the CONICET has decided to promote actions in order taht researchers and teachers have new tools to improve the teaching of science and technology, while promoting scientific vocations in youth. This also provide an innovative way to understand that astronomical phenomena are part of everyday life. And that science can be a center of interest for both teens and society in general.

Our country will host an interesting astronomical event. On February 26 Patagonia will witness its first Annular Eclipse in this century. Later, in the years 2019, 2020, 2024, 2027 and 2034 additional total or annular solar eclipses will be recorded in the country.

This text, written by Professor Dr. Rosa M. Ros and Dr. Beatriz Garcia, president and vice president respectively of the Network for Astronomy Education at School -NASE-, of the International Astronomical Union, contains basics concepts about eclipses and will serve for a better and deeper understanding of these phenomena. But also it allows us to go beyond Astronomy and address other fields, such as Mathematics, Physics and Geography, for example. Undoubtedly, a good effort to alow that issues of science can progressively join the daily agenda.

Dr. Alejandro Cecatto
President of the Consejo Nacional de Investigaciones Cientificas and Tecnicas Argentina

Introduction

In this publication we present some models to explain eclipses and their types. As a preliminary concept, phases of the Moon are introduced in order to explain the eclipses of the Sun and Moon.

These eclipses are also used to find distances and diameters in the Earth-Moon-Sun system using the work of Aristarchus and Eratosthenes developed over 2000 years ago.

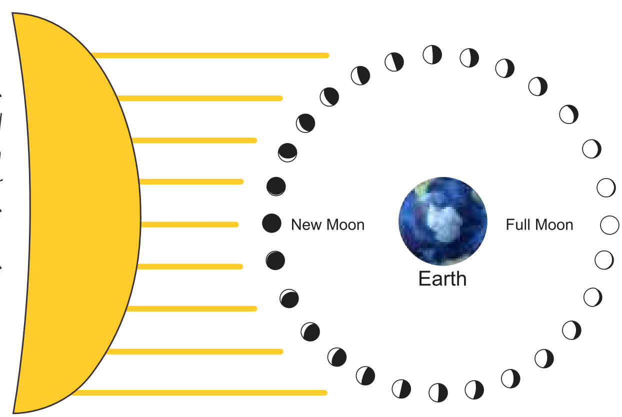
Relative positions

When the relative position of the Earth and the Moon interrupts the passage of sunlight we speak of "eclipse". A solar eclipse happens when the Sun is covered by the Moon when the Moon is located between the Sun and our planet. This kind of eclipse always takes place during new Moon (figure 1a).

Lunar eclipses take place when the Moon crosses the shadow of the Earth. That is when the Moon is on the opposite side of the Sun, so lunar eclipses always occur at full moon (figure 1a).

The Earth and the Moon move along elliptical orbits that are not in the same plane. The orbit of the Moon has an inclination of 5 degrees with respect to the ecliptic (plane of Earth's orbit around the sun). Both planes intersect on a line called the Line of Nodes. The eclipses take place when the Moon is near the Line of Nodes. If both planes coincided, the eclipses would be much more frequent (figure 1b).

Fig. 1a. Solar eclipses take place when the Moon is located between the Sun and the Earth (new Moon). Lunar eclipses occur when the Moon crosses the shadow cone of the Earth (that is, the Earth is located between the Sun and the full Moon).



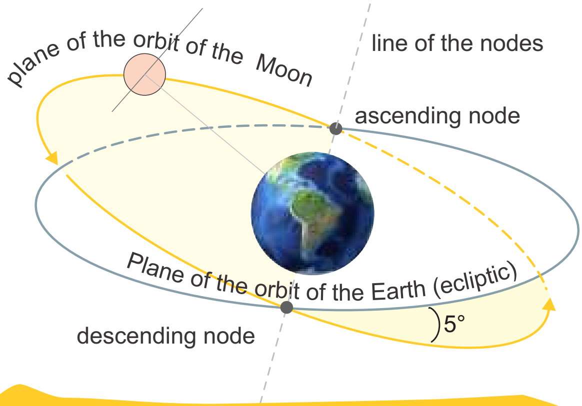


Fig. 1b. Only when the Moon is close to the line of nodes can have an eclipse.

Models with masks for the Moon

Model of Hidden Face

The Moon has a rotational movement and another translational movement, the revolution around the Earth, both of them taking about the same time, about four weeks. For this reason, from the Earth we can only see about half of the lunar surface. (The lunar rotation is 27 days, 7 hours, 43 minutes and 11.5 seconds. The revolution of the Moon around the Earth is 29 days, 12 hours, 44 minutes and 3 seconds. But this durations are as seen from the stars. The Moon's elliptical orbit around the Sun, allows us to see around the sides a bit, with an additional contribution from the tilt of the Moon's orbit, which allows us to see around the top and bottom a bit (for this, we can see 59% of the Moon surface).

Fig. 2a. Model of Hidden Face.

Let's watch it with a simple model. We begin by placing a volunteer who makes as the Earth and a volunteer who acts as the Moon. We will put the lunar volunteer with a round white mask cut out a piece of cardboard. We place the lunar volunteer face to the Earth before moves. Move forward 90° the Moon in its revolution around the Earth, without rotation. Asked the volunteer who is the Earth if he looks the same side of the Moon, that is if he sees the same lunar face and he must see the profile of the volunteer who is the Moon, not the face.

But when the Moon also rotates the same 90 ° in rotation on itself, then the Earth will see the same face always, here only one week has passed. We repeat the process again. We move the Moon 90° again without rotation and happens as before, the Earth does not see the same lunar face, but when he turns other 90° rotation, the Earth sees the same lunar face again with his mask. And so on until a full turn (Figure 2a), corresponding to four weeks. It is clear that the Moon always shows the same face after four weeks and the back of the head of the lunar volunteer is never seen.

Moon Phases model

To explain the phases of the Moon it is best to use a model with a flashlight or with a projector (which will represent the Sun) and a minimum of five volunteers. One of them will be located in the center representing the Earth and the others will situate themselves around "the Earth" at equal distances to simulate different phases of the Moon. To make it more attractive it is a good idea for each "Moon" to wear a white mask that mimics the color of the moon. They should all face the "Earth" because we know that always the Moon offers the same side to the Earth (figure 2).

Fig. 2b. Earth-Moon model with volunteers (to explain the phases and the visible face of the Moon).

We will place the flashlight above and behind one of these volunteers, and begin to visualize the phases (as seen from the Earth, that is in the center). It is very easy to discover that sometimes the mask is completely lighted, sometimes only a quarter is lighted and sometimes not at all lighted (because the flashlight "Sun" is behind that "Moon" and its light dazzles the scene).The greater the number of volunteer "Moons, the more phases can be seen.

Earth-Moon Model

It is not so easy to clearly understand the geometry underlying the phases of the moon, and solar and lunar eclipses. For that reason, we propose a simple model in order to facilitate the understanding of all of these processes.
Insert two nails (about 3 or 4 cm) into a 125 cm. piece of wood. The nails should be separated by 120 cm. Two balls whose diameters are 4 and 1 cm should be placed on them (figure 3).

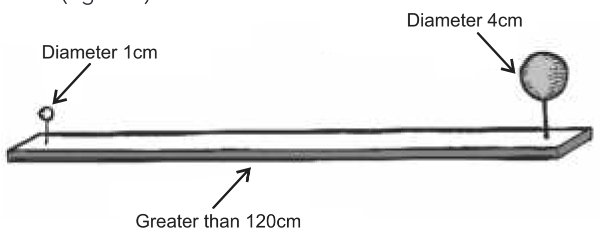


Fig. 3. Earth and Moon model.

It is important to maintain these relative sizes as they represent a scale model of the Earth-Moon system (table 1).

Earth diameter	12800 km.	→	4 cm.
Moon diameter	3500 km.	→	1 cm.
Earth-Moon distance	384000 km.	→	120 cm.
Sun diameter	1400000 km.	→	440 cm. = 4.4 m.
Earth-Sun distance	150000000 km.	→	4700 cm. = 0.47 Km.

Table 1. Distances and diameters of the Earth-Moon-Sun system.

Reproduction of Moon phases

In a sunny place, when the Moon is visible during the day, point the model towards the Moon guiding the small ball towards it (figure 4). The observer should stay behind the ball representing the Earth. The ball that represents the Moon will seem to be as big as the real Moon and the phase is also the same. By changing the orientation of the model the different phases of the Moon can be reproduced as the illumination received from the Sun varies. The Moon-ball has to be moved in order to achieve all of the phases.

It is better to do this activity outdoors, but, if it's cloudy, it can also be done indoors with the aid of a projector as a light source.

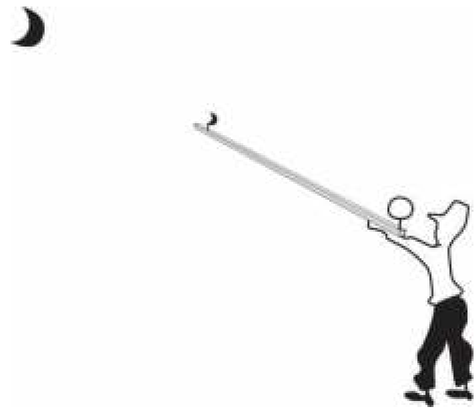


Fig. 4. Using the model in the patio of the school.

Reproduction of lunar eclipses

The model is held so that the small ball of the Earth is facing the Sun (it is better to use a projector to or a flashlight avoid looking at the Sun) and the shadow of the Earth covers the Moon (figure 5a and 5b) as it is larger than the Moon. This is an easy way of reproducing a lunar eclipse. In Figure 6 the Moon is observed across the cone of shadow projected by the Earth in a real moon eclipse.

Fig. 5a and 5b. Lunar eclipse simulation.

Fig. 6. Photographic composition of a lunar eclipse. Our satellite crosses the shadow cone produced by the Earth. Crédit of pictures and copyright: Chander Devgun (SPACE).

Reproducing the eclipses of the Sun

The model is placed so that the ball of the Moon faces the Sun (it is better to use the projector or the flashlight) and the shadow of the Moon has to be projected on the small Earth ball. By doing this, a solar eclipse will be reproduced and a small spot will appear over a region of the Earth (figures 7a, 7b and 8). Figure 9 shows the image of a solar eclipse from space, taken from the International Space Station (ISS).

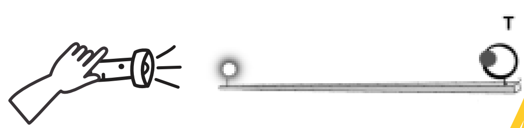


Fig. 7a y 7b. Solar eclipse simulation.

It is not easy to produce this situation because the inclination of the model has to be finely adjusted (that is the reason why there are fewer solar than lunar eclipses).

Fig. 8. Detail of the previous figure 5a.

Fig. 9. Photograph taken from the ISS of the solar eclipse in 1999 over a region of the Earth's surface. Credit picture: ISS.

Fig. 10. Simulating both eclipses.

Observations

- A lunar eclipse can only take place when it is full Moon and a solar eclipse when it is new Moon (figure 1a).
- A solar eclipse can only be seen on a small region of the Earth's surface (figure 8).
- It is rare that the Earth and the Moon are aligned precisely enough to produce an eclipse, and so it does not occur every new or full Moon (figure 1b).

Model Sun-Moon

In order to visualize the Sun-Earth-Moon system with special emphasis on distances, we will consider a new model taking into account the terrestrial point of view of the Sun and the Moon. In this case we will invite the students to draw and paint a big Sun of 220 cm diameter (more than 2 meters diameter) on a sheet (figure 11) and we will show them that they can cover this with a small Moon of 0.6 cm diameter (less than 1 cm diameter).

It is helpful to substitute the Moon ball for a hole in a wooden board in order to be sure about the position of the Moon and the observer.

In this model, the Sun will be fixed 235 meters away from the Moon and the observer will be at 60 cm from the Moon. The students feel very surprised that they can cover the big Sun with this small Moon (figure 12). This relationship of 400 times the sizes and distances is not easy to imagine so it is good to show them with an example in order to understand the scale of distances and the real sizes in the universe.

All these exercises and activities help them (and maybe us) to understand the spatial relationships between celestial bodies during a solar eclipse. This method is much better than reading a series of numbers in a book.

Earth Diameter	12 800 km	2.1 cm
Moon Diameter	3 500 km	0.6 cm
Distance Earth-Moon	384 000 km	60 cm
Sun Diameter	1400 000 km	220 cm
Distance Earth-Sun	150 000 000 km	235 m

Table 2. Distances and diameters of system Earth-Moon-Sun.

Fig. 11. Sun model.

Fig. 12. Observing the Sun and the Moon in the model.

Umbra and penumbra areas and types of eclipses

Although there is an umbra area in the models presented previously, it is not perceived in detail. There is also a penumbra (figures 13 and 14). Umbra means total darkness, while the penumbra is the partial shade between the entirely dark spaces and illuminated entirely generated during eclipses.

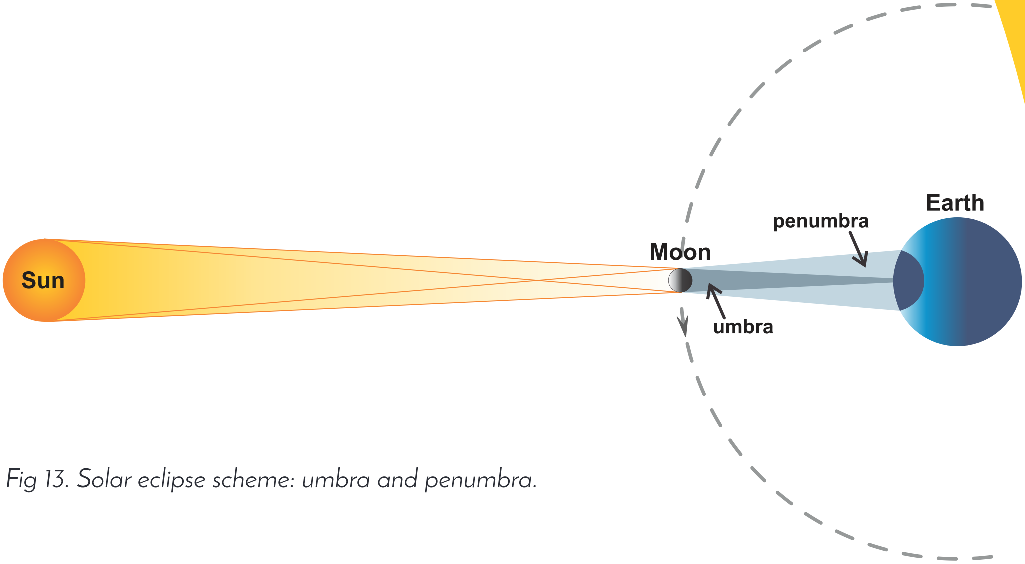


Fig 13. Solar eclipse scheme: umbra and penumbra.

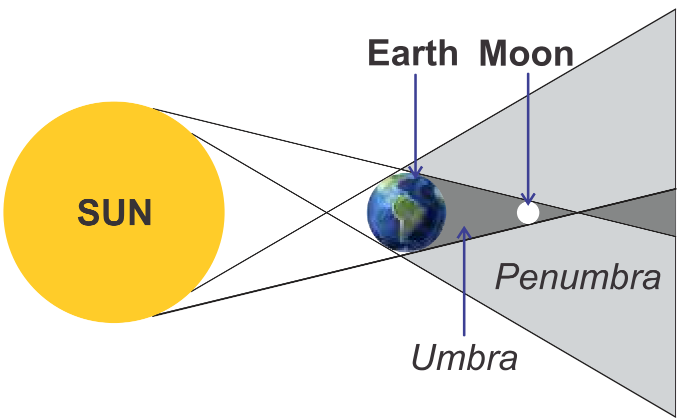


Fig 14. Lunar eclipse scheme: umbra and penumbra.

As a result of an eclipse zones you can classify different types of eclipses.

There are three types of solar eclipse (figure 15):

- Partial: the Moon does not completely cover the solar disk.
- Total: from a strip (full band) on the surface of the Earth, the Moon completely covers the Sun. Out of the band of totality, the eclipse is partial. A total eclipse is for observers located on Earth who are within the lunar shadow cone whose maximum diameter on the surface of our planet does not exceed 270 km. The duration of the total phase can last several minutes, between and 7.5, and the whole phenomenon reaches just over two hours. Annular eclipses have the maximum duration of 12 minutes. Partial eclipses can last about four hours.
- Annular: occurs when the Moon is near apogee (farthest point from the Earth) and its angular diameter is smaller than the Sun, so that even at the maximum phase the visible ring of the disk of the Sun remains visible; outside this zone, the eclipse is partial.

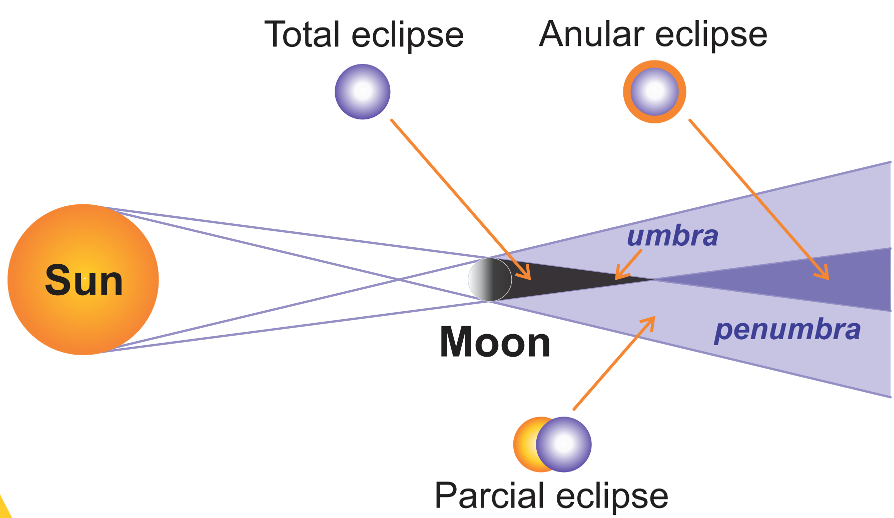


Fig 15. Types of solar eclipses.

There are two types of lunar eclipse (figure 16):

- Partial: The Earth's shadow cone does not completely cover the lunar disk.
- Total: The Earth's shadow cone completely covers the disk of the moon. The duration of the total phase may be from 1.5 to 3.5 hours.

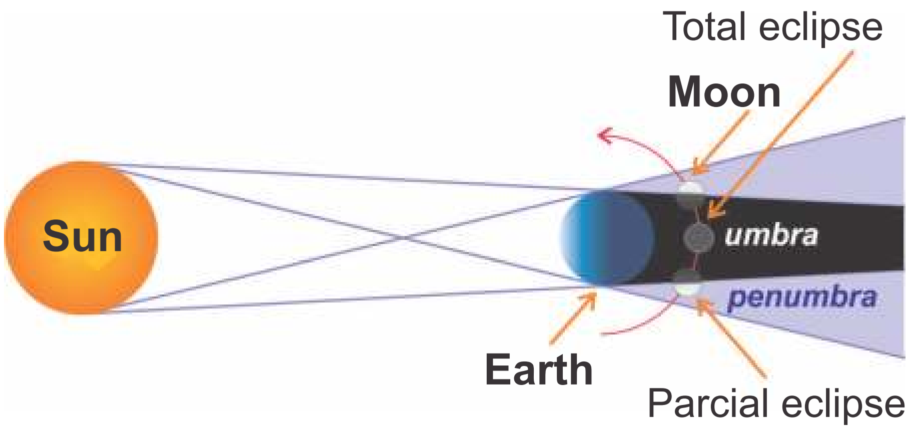


Fig. 16. Types of lunar eclipses.

Eclipses of the Sun for the coming decades

Solar eclipses, as mentioned before, are difficult to observe and that is why we include the distribution of them over some decades (figures 17 and 18).

In addition to the total and annular eclipses another type of eclipse is really very rare: the hybrid eclipses. This phenomenon is very special and corresponds to a combination of total and annular eclipse. In some regions of Earth, this type of eclipse takes on the appearance of an annular eclipse, while from others it can appear as total.

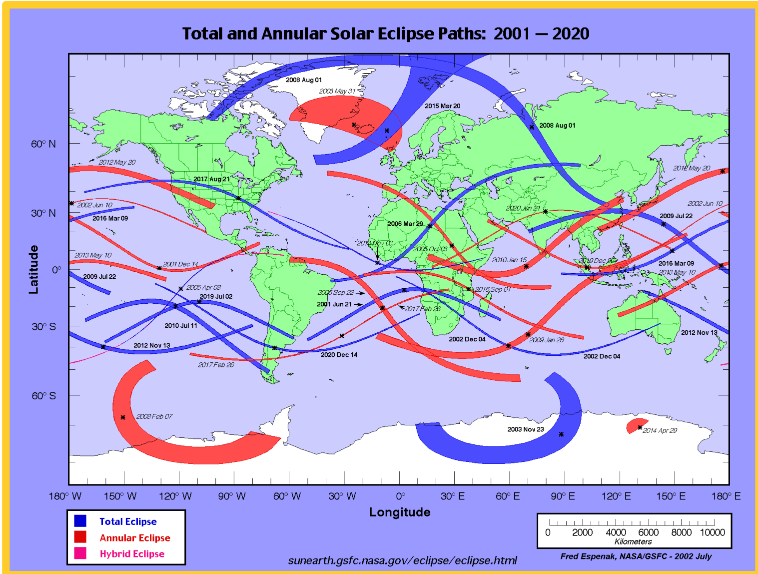


Fig 17. Trajectories of total and annular solar eclipses from 2001 to 2020.

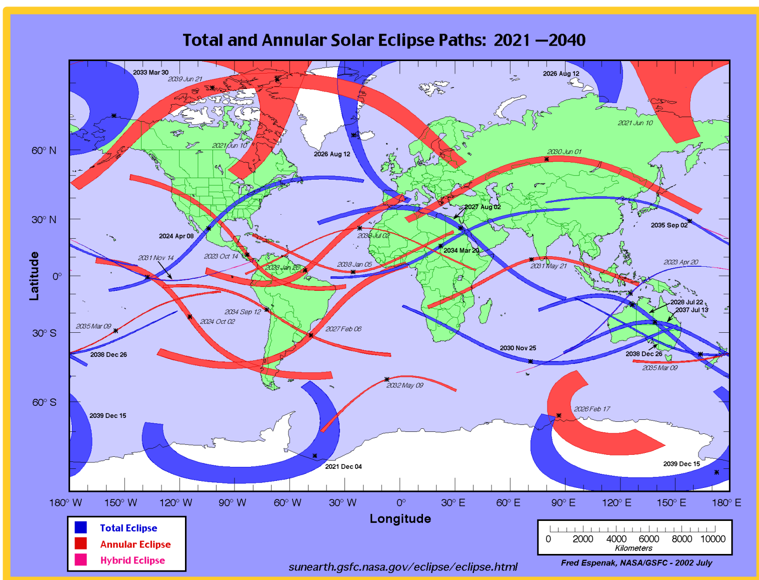


Fig 18. Trajectories of total and annular solar eclipses from 2021 to 2040.

Sizes and Distances in the Earth-Moon-Sun system

Aristarchus (310-230 BC) deduced the proportion between the distances and radii of the Earth-Moon-Sun system. He calculated the radius of the Sun and Moon, the distance from the Earth to the Sun and the distance from the Earth to the Moon in relation to the radius of the Earth. Some years afterwards, Eratosthenes (280-192 BC) determined the radius of our planet and it was possible to calculate all the distances and radii of the Earth-Moon-Sun system. The proposal of this activity is to repeat both experiments as a student activity. The idea is to repeat the mathematical process and, as closely as possible, the observations designed by Aristarchus and Eratosthenes.

It should be mentioned that the work of both scientists have been described using the current mathematical language so that the reader can easily follow.

Aristarchus's experiment again

Aristarchus made diverse observations, performed in different situations with simple tools and little means. We propose new observations and calculations with a group of students. Aristarchus followed for steps to establish relations between distances and diameters, follow in their footsteps:

- Distances Earth-Moon and Earth-Sun.
- Lunar Radius and solar Radius.
- Earth-Moon distance and lunar Radius or Earth-Sun distance and solar Radius.
- Earth's shadow cone and lunar Radius.
- Relate all and leave all values indicated in relation to the radius of the Earth that after using the method of Eratosthenes is calculated.

Relationship between the Earth-Moon and Earth-Sun distances

Aristarchus determined that the angle between the Moon-Sun line and the Earth-Sun line when the moon is in quarter phase is = 87°. Nowadays we know that he was slightly wrong, possibly because it was very difficult to determine the precise timing of the quarter moon. In fact this angle is = 89° 51', but the process used by Aristarchus is perfectly correct.

$$\cos \alpha = \frac{EM}{ES}$$

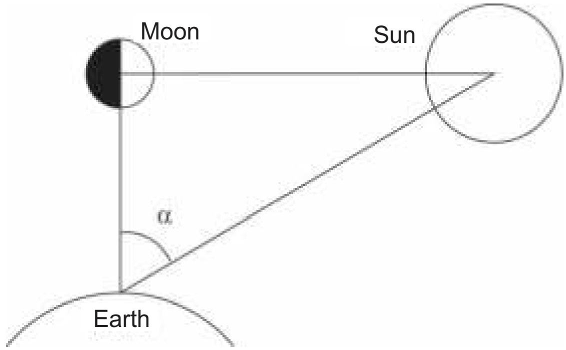


Fig. 19. Relative position of the Moon in quarter phase.

where ES is the distance from the Earth to the Sun, and EM is the distance from the Earth to the moon. Then approximately,

$$ES = 400\ EM$$

(although Aristarchus deduced ES = 19 EM). When this observation with students and a simple Horizontal goniometer is performed, the results are even worse than those of Aristarchus, as the angle to be measured, compared with the right angle, is of a few arcmin.

Relationship between the radius of the Moon and the Sun

The relationship between the diameter of the Moon and the Sun should be similar to the formula previously obtained, because from the Earth we observe both diameters as 0.5°. So both ratios verify

$$R_S = 400\ R_M$$

Relationship between the distance from the Earth to the Moon and the lunar radius or between the distance from the Earth to the Sun and the solar radius

Aristarchus supposes the orbit of the Moon as a circle around the Earth. Since the observed diameter of the Moon is 0.5 degrees, the circular path (360°) of the Moon around the Earth would be 720 times the diameter. The length of this path is 2 times the Earth-Moon distance, i.e. 2 R_M 720 = 2 EM. Solving, we find,

$$EM = \frac{720 R_M}{\pi}$$

Using similar reasoning, since the diameter of the sun is at an angle of half a degree from Earth, the distance from Earth to the sun can be related to the radius of the sun.

$$ES = \frac{720R_S}{\pi}$$

Relation between the distances to the Earth from the Sun and the Moon, and the Moon, Sun and Earth radius

During a lunar eclipse, Aristarchus observed that the time required for the Moon to cross the Earth's shadow cone was twice the time required for the Moon's surface to be covered (figures 20a and 20b). Therefore, he concluded that the shadow of the Earth's diameter was twice the diameter of the Moon, that is, the ratio of both diameters or radius was 2:1. Today, it is known that this value is 2.6:1. Today with digital watches, students get some excellent results, much better than that of Aristarchus, who had no such advantage.

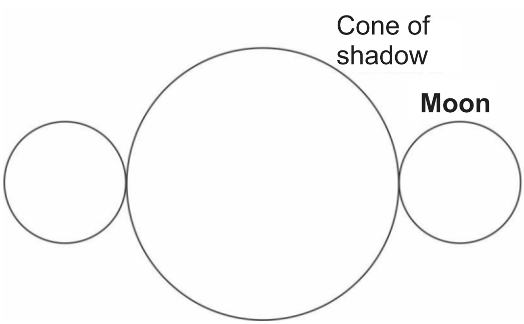


Fig. 20a. Measuring the cone of shadow.

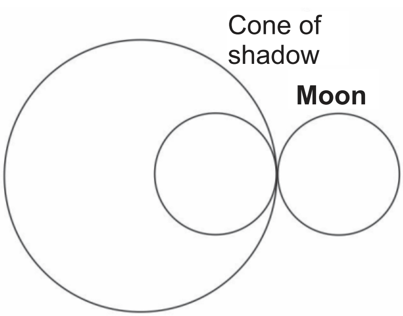


Fig. 20b. Measuring the diameter of the Moon.

Final Formulation

Taken the last result into account, (figure 21) we deduce the following relationship taking x as an auxiliary variable which is then removed.

$$\frac{x}{2.6R_M} = \frac{x+EM}{R_E} = \frac{x+EM+ES}{R_S}$$

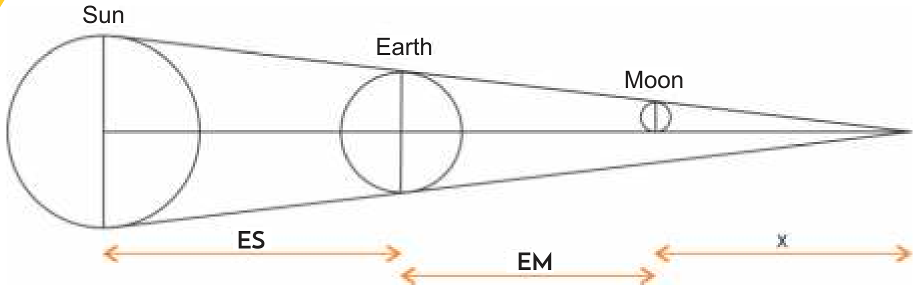


Fig. 21. Shadow cone and relative positions of the Earth-Moon-Sun system.

Introducing into this expresion the relationships ES = 400 EM y R_S = 400 R_M, we can delete x and after simplifying we obtain

$$R_M = \frac{401}{1440} R_E$$

This allows us to express all the sizes mentioned previously as a function of the Earth's radius, so

$$R_S = \frac{2005}{18} R_E \qquad ES = \frac{80200}{\pi} R_E \qquad EM = \frac{401}{2\pi} R_E$$

where we only have to substitute the radius of our planet to obtain all the distances and radio of the Earth-Moon-Sun system.

After these studies it shows that the Sun is much larger than Earth and the Moon, and it is hardly justifiable to think that a large object to rotate relative to one much lower. In fact we know, by later authors, that Aristarchus was driving a heliocentric model with the Sun at the center and Earth spinning around in a circular orbit.

Using the value currently used for the terrestrial radius, R_E = 6378 km, with this initial value we can deduce the diameters and distances following Aristarchus: R_M = 1776 km (real 1738 km), EM = 408 000 km (real 384 000 km), Rs = 740 000 km (real 696 000 km) y ES = 162 800 000 km (real 149 680 000 km). All are of the same order of magnitude as actual values.

Eratosthenes' experiment, again

Eratosthenes was the director of the Alexandrian Library. In one of the texts of the library, he read that in the city of Syene (now Aswan) the day of the summer solstice, the solar noon, the Sun was reflected in the bottom of a well or, what it is the same, a vertical stick on the surface of Earth did not produce shadow. He noted that the same day, at the same time, a stick produced a shadow in Alexandria. From this, he deduced that the surface of the Earth could not be flat, but it should be a sphere (figure 22a and 22b).

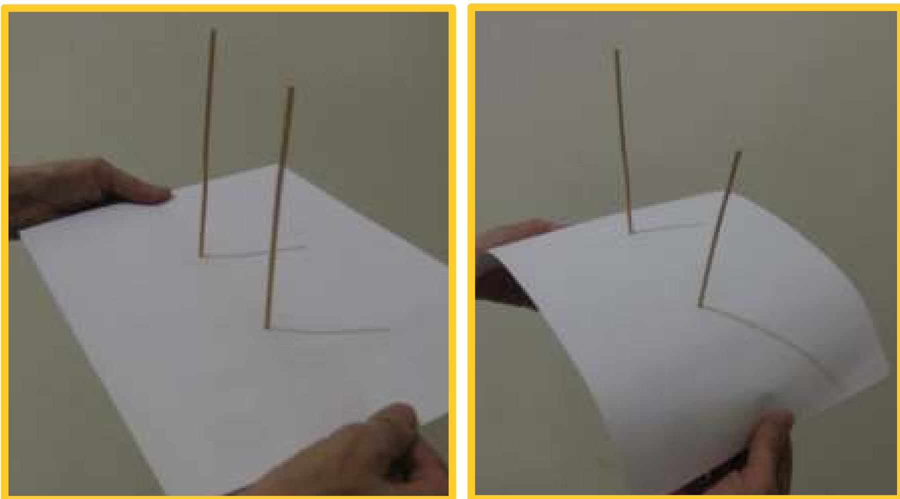


Fig. 22a y 22b. In the flat surface the two sticks produce the same shadow (left), but when the surface is corved sahdows are differetn (right).

Consider two stakes placed perpendicular to the ground, in two cities on the Earth's surface on the same meridian (figure 23). We assume that the solar rays are parallel (as the Sun is very far from Earth). The solar rays produce two shadows, one for each stick. It is sufficient to measure at the same time the length of the shadow of each stick and divide by the length to obtain the angle between the Sun's rays with each stick (angles and respectively in figure 23) using the definition of tangent.

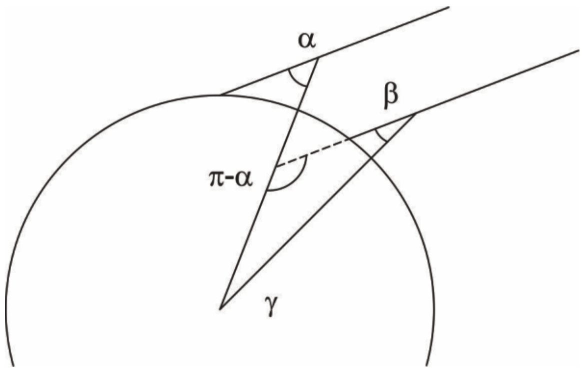


Fig. 23. Placement of plumb-lines and angles in the Eratosthenes experiment.

The central angle can be calculated imposing that the sum of the three angles of the triangle (figure 23) is equal to radians. Then and simplifying

$$\gamma = \alpha - \beta$$

where and have been obtained by the plumb-line and its shadow.

Finally establishing a proportionality between the , the length of its arc d (determined by the distance above the meridian between the two cities), and 2 radians of the meridian circle and its length 2 R_E, we find

$$\frac{2\ \ R_E}{2\pi} = \frac{d}{\gamma}$$

then we deduce that

$$R_E = \frac{d}{\gamma}$$

where has been obtained by the observation and d is the distance in km between both cities. We can find d from a good map. If you can not make the experience a city on the same meridian, it is good to try to do with a city that is as close as possible to the meridian.

In the case of Eratosthenes, the angle was null and , and simply as the distance from Alexandria to Syene was known as caravan route, it was able to deduce the radius of the Earth giving a result very close to the correct one.

As an example, let's look at the results obtained by a group of high school students of Barcelona and Ripoll (Spain). Both cities are on the same meridian but not very far apart. It is better to work with greater distances to gain accuracy, but the results obtained in this case, are not bad. The angles obtained in the two cities were = 0.5194 radians, = 0.5059 radians and the difference = 0.0135 radians. Knowing the distance, in a straight line, on the map between the two cities was d = 89.4 km, we deduce R_E = 6600 km (when the actual value is 6378 km).

The objective of these activities is not the accuracy of the results. Instead, we want that students have an example to show the results you are able to obtain using the knowledge they have accumulated throughout their training and a little ingenuity.

Eratosthenes international project

As it mentioned two observers to measure the Earth's radius suffice, but any school can be added to projects that are organized in this respect. It notes that for over ten years the Buenos Aires University, Argentina, in collaboration with the Library of Alexandria, the European Association for Astronomy Education, EAAE, and various institutions of more than 20 countries organise an "Eratosthenes project" that includes more than 100 schools. In this case the mathematical development is much more complex and does not respond to secondary mathematical content, but it is certainly very interesting to send the data obtained in this project by allowing contact with other students other countries.

For more details (information in Spanish, English and Portuguese):
http://df.uba.ar/es/actividades-y-servicios/difusion/proyecto-eratostenes/eratostenes-2016

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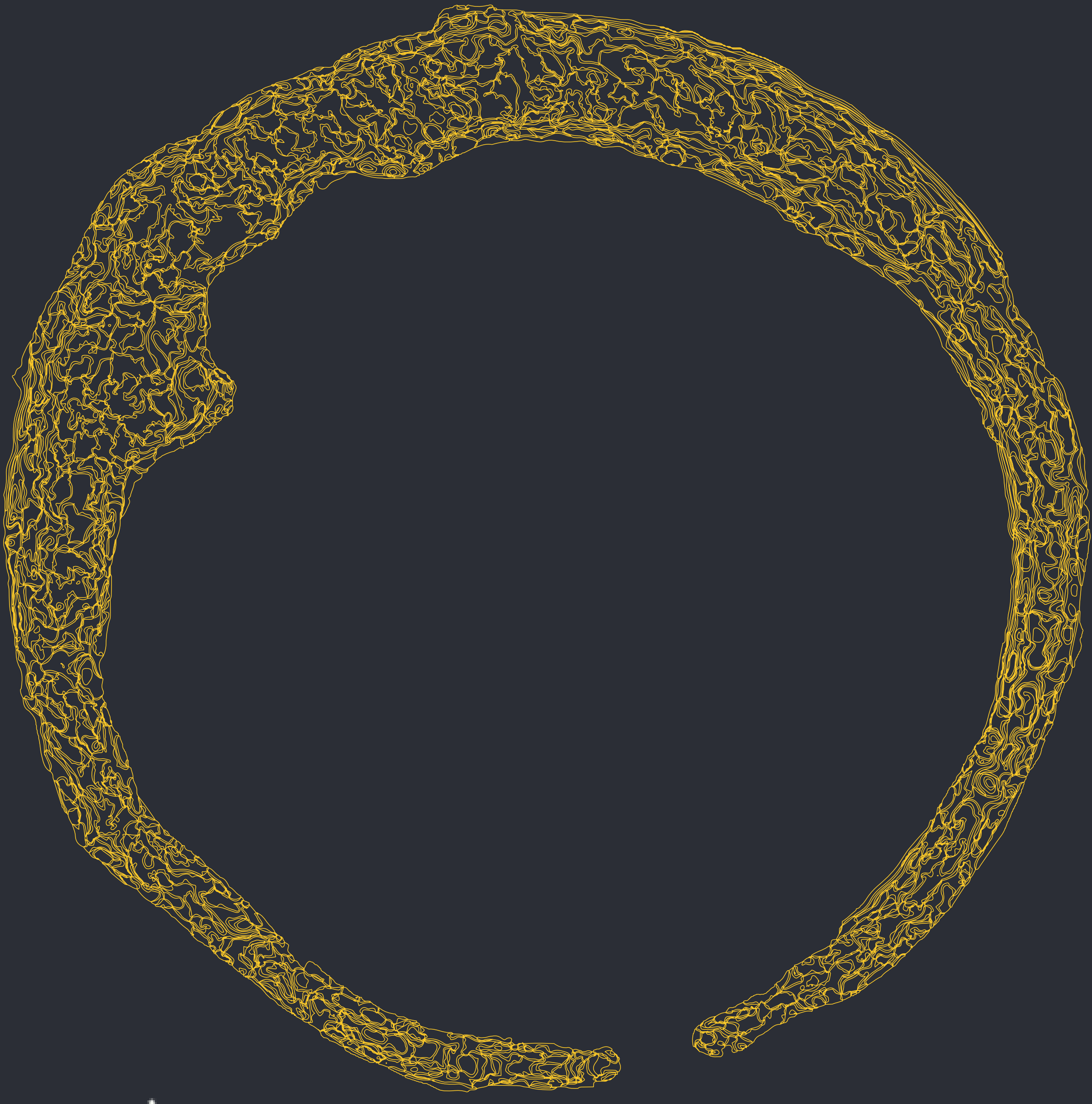

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