Stellar Lives

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Summary

To understand the life of the stars it is necessary to understand what they are, how we can find out how far away they are, how they evolve and what are the differences between them. Through simple experiments, it is possible to explain to students the work done by scientists to study the composition of the stars, and also build some simple models.

Goals

This workshop complements the stellar evolution NASE course, presenting various activities and demonstrations centered on understanding stellar evolution. The main goals are to:

- Understand the difference between apparent magnitude and absolute magnitude.
- Understand the Hertzsprung-Russel diagram by making a color-magnitude diagram.
- Understand concepts such as supernova, neutron star, pulsar, and black hole.

Activity 1: The Parallax Concept

Parallax is a concept that is used to calculate distances in astronomy. We will perform a simple activity that will allow us to understand what parallax is. Face a wall at a certain distance, which has landmarks: wardrobe, tables, doors, etc. Stretch your arm in front of you, and hold your thumb vertically (figures 1a and 1b).

First close your right eye, see the example with the finger on the center of a picture. Without moving your finger, close your right eye and open the left eye. The finger moved, it no longer coincides with the center of the picture but with the edge of the box.

For this reason, when we observe the sky from two distant cities, bodies that are closer, such as the moon, are offset with respect to the background stars, which are much more distant. The shift is greater if the distance between the two places where observations are taken is farther apart. This distance is called baseline.
Fig. 1a: With your arm extended look at the position of your thumb relative to the background object, first with the left eye (closing the right one) and then, Fig. 1b, look with the right eye (with the left eye closed).

Calculation of distances to stars by parallax

Parallax is the apparent change in the position of an object, when viewed from different places. The position of a nearby star relative to background stars that are farther away seems to change when viewed from two different locations. Thus we can determine the distance to nearby stars.

The parallax is appreciable if the distance that is the baseline is maximized. This distance is the diameter of the orbit of the Earth around the sun (figure 2).

Fig. 2: The parallax angle $p$ is the angular shift one sees when observing a star from two locations that are one Earth-Sun distance apart.

Fig. 3: By measuring the parallax angle, $p$, it is then possible to calculate the distance $D$ to the object.
For example if we observe a nearby star with respect to background stars from two positions A and B of the Earth's orbit (figure 3), separated by six months, we can calculate the distance D that the star is at, giving:

$$\tan p = \frac{AB/2}{D}$$

Since p is a very small angle, the tangent can be approximated as the angle measured in radians:

$$D = \frac{AB/2}{p}$$

The base of the triangle AB / 2 is the Earth-Sun distance, 150 million km. If we have the parallax angle p, then the distance to the star, in kilometers, will be $$D = \frac{150,000,000}{p}$$, with the angle p expressed in radians. For example, if the angle p is an arc second, the distance to the star is:

$$D = \frac{150000000}{2\pi/(360 \times 60 \times 60)} = 30939720937064 \text{ km} = 3.26 \text{ a.l.}$$

This is the unit of distance that is used in professional astronomy. If you saw a star with a parallax of one arc second, it is at a distance of 1 parsec (par-sec), equivalent to 1pc = 3.26 light years. A smaller parallax implies a larger distance to the star. The relationship between distance (in pc) and parallax (in arcseconds) is:

$$d = \frac{1}{p}$$

The simplicity of this expression is the reason for which it is used. For example, the closest star is Proxima Centauri, has a parallax of "0.76, which corresponds to a distance of 1.31 pc, equivalent to 4.28 ly. The first parallax observation made of a star (61 Cygni) was made by Bessel in 1838. Although at the time it was suspected that the stars were so distant, that they could not be measured with accurate distances.

Currently, we use parallax to measure distances to stars that are within 300 light years of us. Beyond that distance, the parallax angle is negligible, so we must use other methods to calculate distances. However, these other methods are generally based on comparison with other stars whose distance is known from the parallax method. Parallax provides a basis for other distance measurements in astronomy, the cosmic distance ladder. Parallax is essentially the bottom rung of this distance ladder.

**Activity 2: Inverse-square law**

A simple experiment can be used to help understand the relationship between luminosity, brightness, and distance. It will show that the apparent magnitude is a function of distance. As shown in figure 11, you will use a light bulb and a card (or box) with a small square hole cut out of it. The card with the hole is placed to one side of the light bulb. The light bulb radiates in all directions. A certain amount of light passes through the hole and illuminates a mobile screen placed parallel to the card with the hole. The screen has squares of the same size as the hole in the card. The total amount of light passing through the hole and reaching the screen
does not depend on how far away we put the screen. However, as we put the screen farther away this same amount of light must cover a larger area, and consequently the brightness on the screen decreases. To simulate a point source and reduce shadows, we can also use a third card with a hole very close to the light bulb. However, be careful not to leave that card close to the bulb for too long, as it might burn.

![Fig. 4: Experimental setup](image)

We observe that when the distance between the screen and the light bulb doubles, the area that the light illuminates becomes four times bigger. This implies that the light intensity (the light arriving per unit area) becomes one fourth of the original amount. If the distance is tripled, the area on the screen over which light is spread becomes nine times bigger, so the light intensity will be a ninth of the original amount. Thus, one can say that the intensity is inversely proportional to the square of the distance to the source. In other words, the intensity is inversely proportional to the total area that the radiation is spread over, which is a sphere of surface area $4\pi D^2$.

**The magnitude system**

Imagine a star is like a light bulb. The brightness depends on the power of the star or bulb and distance from which we see it. This can be verified by placing a sheet of paper opposite a lamp: the amount of light that reaches the sheet of paper depends on the power of the bulb, and the distance between the sheet and the bulb. The light from the bulb is spread out evenly across a surface of a sphere, which has an area of $4\pi R^2$, where $R$ is the distance between the two objects. Therefore, if you double the distance ($R$) between the sheet of paper and the bulb (figure 5), the intensity that reaches the paper is not half, but is one-fourth (the area that the light is distributed over is four times higher). And if the distance is tripled, the intensity that reaches the paper is one-ninth (the area of the sphere that the light is distributed over is nine times higher).

The brightness of a star can be defined as the intensity (or flow) of energy arriving at an area of one square meter located on Earth (Fig. 5). If the luminosity (or power) of the star is $L$, then:

$$B = F = \frac{L}{4\pi D^2}$$
Fig. 5: The light becomes less intense the further away it is

Since the brightness depends on the intensity and distance of the star, one can see that an intrinsically faint star that is closer can be observed to be the same brightness as an intrinsically more luminous star but that is farther away.

Hipparchus of Samos, in the second century BC, made the first catalog of stars. He classified the brightest stars as 1st magnitude stars, and the faintest stars as 6th magnitude stars. He invented a system of division of brightness of the star that is still used today, although slightly rescaled with more precise measurements than what was originally made with the naked eye.

A star of magnitude 2 is brighter than a star with a magnitude of 3. There are stars that have a magnitude of 0, and even some stars that have negative magnitudes, such as Sirius, which has a magnitude of -1.5. Extending the scale to even brighter objects, Venus has a visual magnitude of -4, the full moon has a magnitude of -13, and the Sun has a magnitude of -26.8.

These values are properly called apparent magnitudes $m$, since they appear to measure the brightness of stars as seen from Earth. This scale has the rule that a star of magnitude 1 is 2.51 times brighter than a star of magnitude 2, and this star is 2.51 times brighter than another star of magnitude 3, etc. This means that a difference of 5 magnitudes between two stars is equivalent to the star with the smaller magnitude being $2.51^5 = 100$ times brighter. This mathematical relationship can be expressed as:

$$\frac{B_1}{B_2} = (\sqrt[100]{10})^{m_2-m_1}$$

or

$$m_2 - m_1 = 2.5 \log \left(\frac{B_1}{B_2}\right)$$

The apparent magnitude $m$ is a measure related to the flux of light into the telescope from a star. In fact, $m$ is calculated from the flux $F$ and a constant $C$ (that depends on the flow units and the band of observation) through the expression:

$$m = -2.5 \log F + C$$

This equation tells us that the greater the flux, the more negative a star’s magnitude will be. The absolute magnitude $M$ is defined as the apparent magnitude $m$ that an object would have if it was seen from a distance of 10 parsecs.

To convert the apparent magnitude into an absolute magnitude it is necessary to know the exact distance to the star. Sometimes this is a problem, because distances in astronomy are
often difficult to determine precisely. However, if the distance in parsecs $d$ is known, the absolute magnitude $M$ of the star can be calculated using the equation:

$$M = m - 5\log d + 5$$

**The colors of stars**

It is known that stars have different colors. At first glance with the naked eye one can distinguish variations between the colors of stars, but the differences between the colors of stars is even more apparent when stars are observed with binoculars and photography. Stars are classified according to their colors; these classifications are called spectral types, and they are labeled as: O, B, A, F, G, K, M. (figure 6).

According to Wien's law (figure 7), a star with its maximum intensity peaked in blue light corresponds to a higher temperature, whereas if a star’s maximum intensity peaks in the red then it is cooler. Stated another way, the color of the star indicates the surface temperature of the star.

![Spectral Class Types for Stars](image)

*Fig. 6: Spectral Types of Stars, according their colors*

![Wien's Law](image)

*Fig. 7: If the temperature increases, the peak of the star’s intensity moves from the red to the blue.*

**Activity 3: Stellar colors**

First, you will use a simple incandescent lamp with a variable resistor to illustrate blackbody radiation. By placing colored filters between the lamp and the spectroscope, students can
examine the wavelength of light transmitted through the filters. By comparing this to the spectrum of the lamp, students can demonstrate that the filters absorb certain wavelengths. Then, students can use a device similar to that in figure 8a, which has blue, red, and green lights, and is equipped with potentiometers, to understand the colors of stars. This device can be constructed by using lamps, where the tubes of the lamps are made with black construction paper, and the opening opposite the bulb is covered with sheets of colored cellophane. Using this device, we can analyze figure 8b and try to reproduce the effect of stellar temperature rise. At low temperatures the star only emits red light in significant amounts.

If the temperature rises there will also be emission of wavelengths that pass through the green filter. As this contribution becomes more important the star’s color will go through orange to yellow. As temperature rises the wavelengths that pass the blue filter become important and therefore the star’s colors become white. If the intensity of the blue wavelengths continues to grow and becomes significantly greater than the intensities of the wavelengths that pass through the red and green filters, the star becomes blue. To show this last step, it is necessary to reduce the red and green lamp intensity if you used the maximum power of the lamps to produce white.

![Fig. 8a](image1.png) ![Fig. 8b](image2.png)

**Fig. 8a: Device to explain the star color, Fig. 8b: Projection to explain the color of stars and the production of white color.**

**How do we know that stars evolve?**

Stars can be placed on a Hertzsprung-Russell diagram (figure 9a), which plots stellar intensity (luminosity or absolute magnitude) versus stellar temperature or color. Cool stars have lower luminosity (bottom right of the plot); hot stars are brighter and have higher intensity (top left of the plot). This track of stars that forms a sequence of stars from cool temperature / low luminosity up to high temperature / high luminosity is known as the Main Sequence. Some stars that are more evolved have “moved off” of the main sequence. Stars that are very hot, but have low luminosity, are white dwarfs. Stars that have low temperatures but are very bright are known as supergiants.

Over time, a star can evolve and "move" in the HR diagram. For example, the Sun (center), at the end of its life will swell and will become a red giant. The Sun will then eject its outer layers and will eventually become a white dwarf, as in figure 9b.
Fig. 9a: H-R Diagram. Fig. 9b: The Sun will shed its external atmosphere and will convert into a white dwarf, like that which exists in the center of this planetary nebula.

**Activity 4: The age of open clusters**

Analyze the picture (figure 10) of the Jewel Box cluster or Kappa Crucis, in the constellation of the Southern Cross.
It is obvious that the stars are not all the same color. It is also difficult to decide where the cluster of stars ends. On figure 10, mark where you think the edge of the cluster is.

In the same figure 10, mark with an "X" where you think the center of the cluster is. Then, use a ruler to measure and draw a square with a side of 4 cm around the center. Measure the brightness of the star closest to the upper left corner of your square, based on its size compared with the comparison sizes that are presented in the guide on the margin of Figure 4. Estimate the color of the star with the aid of the color comparison guide located on the left side of figure 10. Mark with a dot the color and size of your first star on the color-brightness worksheet (figure 11).

Note that color is the x-axis while brightness (size) is the y-axis. After marking the first star, proceed to measure and mark the color and brightness (size) of all the stars within the square of 4 cm.

The stars of the Jewel Box cluster should appear to follow a certain pattern in the graph you have created in figure 11. In figure 10, there are also stars that are located in front and behind the cluster but are not actually a part of it. Astronomers call them “field stars”. If you have time, try to estimate how many field stars you have included in the 4 cm square that you used for your analysis, and estimate their color and brightness. To do this, locate the field stars in the color-magnitude diagram and mark them with a tiny “x” instead of a dot. Note that the field stars have a random distribution on the graph and don’t seem to form any specific pattern.

Most of the stars are located on a strip of the graph that goes from the top left to the bottom right. The less massive stars are the coldest ones and appear red. The most massive stars are the hottest and brightest, and appear blue. This strip of stars on the color-magnitude diagram is called the “main sequence”. Stars on the main sequence are placed in classes that go from the O class (the brightest, most massive, and hottest: about 40,000 K) to the M class (low brightness, low mass, and small stellar surface temperature: about 3500 K).

During most of the life of a star, the same internal forces that produce the star’s energy also stabilize the star against collapse. When the star runs out of fuel, this equilibrium is broken and the immense gravity of the star causes it to collapse and die.

The star’s transition between life on the main sequence and collapse is a part of the stellar cycle called the “red giant” stage. Red giant stars are bright because they have stellar diameters that can go from 10 to more than 300 times larger than the Sun. Red giants are also red because their surface temperature is low. In the worksheet they are classified as K or M stars but they are very bright. The most massive stars exhaust their fuel faster than lower-mass stars and therefore are the first to leave the main sequence and become red giants. Because of their large sizes that can be more than 1000 Sun diameters, the red giants with masses between 10 and 50 solar masses are called “red supergiants” (or red hypergiants if they came from an O class star). Red giants expand and cool down, becoming red and bright, and are therefore located in the top right of the color-magnitude diagram. As the cluster gets older, the amount of stars that leave the main sequence to become red giants grows. Therefore, the age
of a star cluster can be determined by the color of the biggest and brightest star that still remains on the main sequence.

Many stars in old clusters have evolved beyond the stage of red giants to another stage: they become white dwarfs. White dwarfs are very small stars that are about the size of the Earth. They are also very faint, and therefore cannot be seen in this image of the Jewel Box. Can you estimate an age for the Jewel Box star cluster from your graph in figure 11 by comparing to the graphs of star clusters of different ages shown in figures 12a, 12b and 12c?

If you understand the HR diagram and the relationship between color (surface temperature), brightness, and ages of stars, it is possible to explain how stars and star clusters evolve. You can compare the lives of O/B class stars with those of A/F/G and K/M stars. You can see that stars of the same mass evolve in the same way even in different star clusters. Because of this, you can see the differences in ages between different star clusters using the HR diagram. This is why you can tell that figure 12a shows a young cluster (it has O and B stars in the main sequence and we know that these stars rapidly evolve to red supergiants), and that figure 12c shows an old cluster (with almost only K/M stars in the main sequence and many stars in the red giant phase).

We can ask ourselves: “What would the Sun’s position be in the Hertzsprung-Russell diagram?” The Sun is a star with a surface temperature of 5870 K and therefore it appears yellow. This would correspond to a G2 class (x-axis). It is in the main sequence stage of its evolution, where hydrogen is being fused into helium in the stellar core. This puts it in class 5 of luminosity, along with many other stars located on the main sequence.

**Stellar death**

The end of a star’s life depends on the mass of the progenitor star, as is shown in figure 13. At a certain point in the evolution of star clusters the more massive stars disappear from the Hertzsprung-Russell diagram. While the low mass stars will evolve into white dwarfs, these
massive stars will end their lives as one of the most violent phenomena in the universe: supernovae. The remains of these kinds of phenomena will be objects that have no thermal emission (pulsars and black holes) and therefore are not visible in the Hertzsprung-Russell diagram.

![Evolution of Stars Diagram](image)

**Fig. 13:** Evolution of stars according their masses.

**What is a supernova?**

Is the dead of a massive star. The stellar main sequence is characterized by the fusion of hydrogen to produce helium, subsequently progressing to the production of carbon and increasingly heavier elements. The final product is iron. The fusion of iron is not possible because this reaction would require energy to proceed instead of releasing energy.

The fusion of different elements proceeds until the supply of that element is exhausted. This fusion happens outwards from the core, so after time the star acquires a layered structure somewhat like an onion (figure 14b), with heavier elements in the deeper layers near the core.

A 20 Solar mass star has these stages:
- 10 million years burning Hydrogen in the core (main sequence)
- 1 million years burning Helium
- 300 years burning Carbon
- 200 days burning Oxygen
- 2 days to consume Silicon: the supernova explosion is imminent.

When the star finally has an iron core, no further nuclear reactions are possible. Without radiation pressure from fusion to balance gravity, the star’s collapse is unavoidable, without
the possibility of any further nuclear ignition. During the collapse, atomic nuclei and electrons are pushed together to form neutrons and the central part of the core becomes a neutron star.

![Fig. 14a: Remains of a supernova](image1)

![Fig. 14b: Layer structure of the interior of a star before exploding into a supernova](image2)

Neutron stars are so dense that a teaspoonful would weigh as much as all the buildings in a large city. As neutrons are squeezed together, no further contraction that can take place. Particles infalling from the outer layers of the star at speeds of about a quarter of the speed of light hit the neutron core and are suddenly stopped. This causes them to bounce back in the form of a shock wave, resulting in one of the most energetic processes known in the universe (Fig 14a): single exploding star can outshine an entire galaxy consisting of billions of stars.

During this rebound the energies are so large that some elements heavier than iron are created (such as lead, gold, uranium, etc.). These elements emerge violently during the explosion and are ejected along with all the outer matter of the star. In the center of the ejected material there remains a neutron star spinning at high speed, or if the original star was massive enough, a black hole.

**Activity 5: Simulation of a supernova explosion**

When a star explodes as a supernova, the light atoms in the outer layers fall toward the heavier elements in the interior and finally bounce off the solid central core.

![Fig. 15: We dropped at the same time both a tennis ball and a basketball.](image3)
A simplified model of this process can be represented in an easy and rather spectacular way with a basketball and a tennis ball, by dropping them together onto a hard surface such as the floor (figure 15). In this model, the floor represents the dense stellar core, the basketball represents a heavy atom that bounces back from the core and pushes the light atom right behind it, represented by the tennis ball.

To present the model, hold the basketball at eye level with the tennis ball just above it, as vertical as possible. Drop the two balls together. You might guess that the balls would rebound to the same height from which they started, or maybe even lower because of friction and energy dissipation to the floor. However, the result is quite surprising.

When you drop the two balls, they arrive almost simultaneously to the floor. The big ball bounces elastically back nearly at the same speed it had when it reached the floor. At that moment it collides with the little tennis ball that was falling with the same speed as the basketball. The tennis ball bounces off the basketball at high speed and reaches much higher than the height from which the balls were dropped. If this experiment were repeated, using a large number of even lighter balls, their rebound speeds would be fantastic.

In the model presentation, the tennis ball rebounds to twice the original height from which the two balls were dropped. In fact, be careful not to break something if you do this experiment indoors.

This experiment can be done in the classroom or in another enclosed area, but preferably it should be done outdoors. It can be done from a high window, but this will make it difficult to make sure the balls drop vertically and the balls can bounce with great force in unpredictable directions.

Some toy stores or science museum shops sell a toy called the "Astro Blaster" that is based on the same principle. It consists of four small rubber balls of different sizes linked by an axis. The smaller balls shoot into the air, rebounding after the system hits the ground.

**What is a neutron star?**

A neutron star is the remnant of a very massive star that has collapsed and has shed its outer layers in a supernova explosion. Neutron stars are usually no bigger than a few dozen kilometers. As the name implies, they consist of neutrons stacked together to an incredible density: a single thimble of this matter would weigh millions of tons.

A neutron star forms if the remnant of a supernova is between 1.44 and about 8 solar masses.

**What is a pulsar?**

A pulsar is a neutron star spinning at extremely high speed (figure 16). When a massive star collapses, the outer layers fall toward the core and start spinning faster due to conservation of angular momentum. This is similar to how a skater spins faster by drawing her arms toward her body.
The star's magnetic field generates strong electromagnetic synchrotron emission in the direction of its axis. However because the magnetic field axis does not usually coincide with the axis of rotation, (as is also the case on Earth) the rotating neutron star acts like a giant cosmic lighthouse. If this emission happens to be directed toward the Earth, we detect a pulse at regular intervals.

In 1967, Bell and Hewish discovered the first pulsar. The pulse signal came from a point in space where nothing was observed pulsing in visible light. The rapid pulse repetition was striking - several times per second with amazing precision.

At first it was thought that pulsars could be intelligent extraterrestrial signals. Then more pulsating radio sources were discovered, including the center of the Crab Nebula. Scientists knew that this nebula was produced by a supernova and could finally explain the origin of pulsars. The pulsar PSR B1937+21 is one of the fastest known pulsars and spins over 600 times per second. It is about 5 km in diameter and were it spinning about 10% faster, it would be broken apart by the centrifugal force. Hewish won the Nobel Prize in 1974.

Another very interesting pulsar is a binary system called PSR 1913+16 in the Eagle constellation. The mutual orbital motion of the stars in a very intense gravitational field produces some slight delays in the emissions we receive. Russell Hulse and Joseph Taylor have studied this system and confirmed many predictions of the theory of relativity, including the emission of gravitational waves. These two Americans were awarded the Nobel Prize in 1993 for their research.

**Activity 6. Pulsar simulation**

A pulsar is a neutron star that is very massive and spinning quickly. It emits radiation but the source is not fully aligned with the axis of rotation, so the emitted beam of radiation spins like
a lighthouse. If this beam is oriented toward Earth, we observe a radiation pulse several times per second.

We can simulate a pulsar with a flashlight (figure 17a) tied with a rope to the ceiling. If we turn it on and spin it (figure 17b), we will see light intermittently whenever the flashlight is pointing in our direction (figure 17c).

If you tilt the flashlight so that it is not horizontal, you will no longer be able to see the beam of light from the same position. Therefore, we can only observe the emission of a pulsar if we are well aligned with its rotation.

What is a black hole?

If we throw a stone upwards, gravity slows it down until it returns back to the ground. If we throw the stone with a larger initial speed, the stone goes higher before it falls back down. If the initial speed is 11 km/s, the escape velocity of Earth, the stone would not fall back down (assuming there is no air friction).

If Earth collapsed while maintaining its mass, the escape velocity at its surface would increase because we would be closer to the center of the Earth. If it collapsed to a radius of 0.8 cm, the escape velocity would become greater than the speed of light. Since nothing can exceed the speed of light, nothing would be able to escape from the surface, not even light. The Earth would have become a black hole the size of a tiny marble.

Theoretically, it is possible for black holes to have very small masses. In reality however, there is only one known mechanism that can concentrate mass to the necessary densities: gravitational collapse. In order for gravitational collapse to take place, a very large amount of mass is needed. We learned that neutron stars are the remnants of stars of mass 1.44 to about 8 solar masses. However, if the original star is even more massive, gravity is so strong that its interior may continue collapsing until it becomes a black hole. Therefore, this type of black hole will have a mass several times larger than our Sun. Black holes’ densities are very impressive. A tiny marble made of matter this dense would weigh as much as the whole Earth.
Although we cannot observe them directly, we know of several candidates for black holes in the universe through the emission from material revolving around the black hole at high speeds. For example, right in the center of our galaxy we see nothing, but we can detect a ring of gas swirling around the center at incredible speed. The only possible explanation is that there is a huge invisible mass at the center of this ring, weighing as much as three or four million suns. This can only be a black hole, with a Schwarzschild radius slightly larger than our Sun. These types of black holes, which are located at the centers of many galaxies, are called supermassive black holes.

Activity 7. Simulation of space curvature and a black hole

It's easy to simulate the two-dimensional curvature of space created by a black hole using a piece of elastic fiber sheet called Lycra (figure 18) or a large piece of gauze.

First, stretch the fiber sheet or mesh. Now, roll a small ball (or marble) along the sheet. This represents a photon of light and its trajectory simulates the straight path of a light ray in the absence of curvature. However, if you place a heavy ball at the center of the sheet and then roll the smaller ball (or marble) its path will follow a curve. This simulates the path of a light ray in a curved space caused by the presence of a gravitating mass. How much the path of the light ray curves depends on how close the light beam passes to the gravitating body and how massive this body is. The angle of deflection is directly proportional to the mass and inversely proportional to the distance. If we loosen the tension in the sheet, it simulates a deeper gravity well, which makes it more difficult for the smaller ball to leave. It becomes a model of a black hole.
Bibliography