Earth-moon-sun system: Phases and eclipses

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Summary

The following work deals with moon phases, solar eclipses, and lunar eclipses. These eclipses are also used to find distances and diameters in the Earth-Moon-Sun system.

Finally, the origin of tides is also explained.

Goals

- To understand why the moon has phases.
- To understand the cause of lunar eclipses.
- To understand why solar eclipses occur.
- To determine distances and diameters of the Earth-Moon-Sun system.
- To understand the origin of the tides.

Relative positions

The term “eclipse” is used for very different phenomena, but in all cases an eclipse takes place when one object crosses in front of another object; for this unit, the relative positions of the Earth and the Moon (opaque objects) cause the interruption of sunlight.

A solar eclipse happens when the Sun is covered by the Moon when it is located between the Sun and our planet. This kind of eclipse always takes place during new Moon (figure 1).

Lunar eclipses take place when the Moon crosses the shadow of the Earth. That is when the Moon is on the opposite side of the Sun, so lunar eclipses always occur at full moon phase (figure 1).

The Earth and the Moon move along elliptical orbits that are not in the same plane. The orbit of the Moon has an inclination of 5 degrees with respect to the ecliptic (plane of Earth's orbit around the sun). Both planes intersect on a line called the Line of Nodes. The eclipses take place when the Moon is near the Line of Nodes. If both planes coincided, the eclipses would be much more frequent than the zero to three times per year.

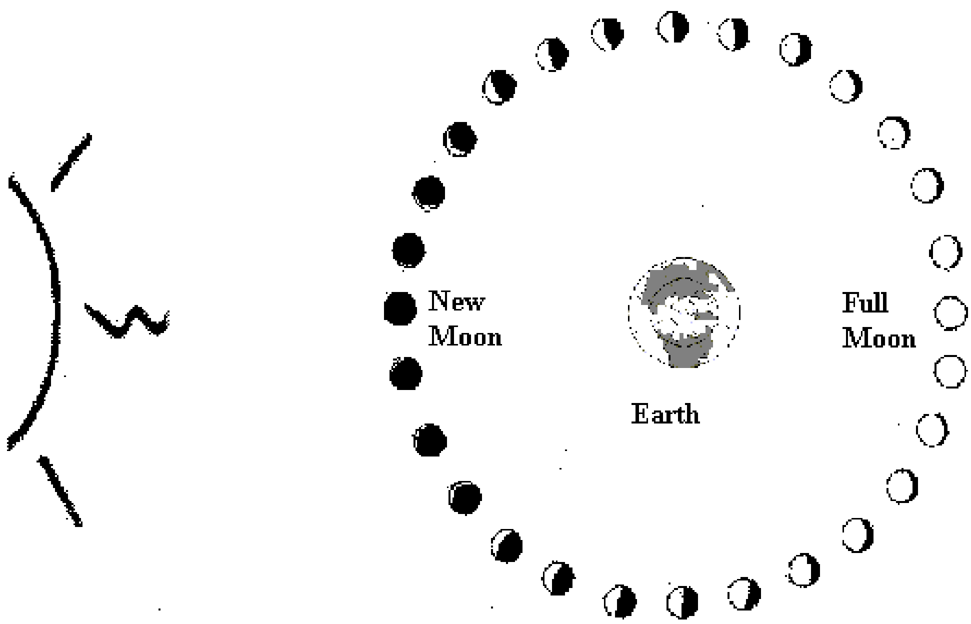


Fig.1: Solar eclipses take place when the Moon is located between the Sun and the Earth (new Moon). Lunar eclipses occur when the Moon crosses the shadow cone of the Earth (that is, the Earth is located between the Sun and the full Moon).

Masks models

Model of Hidden Face

The Moon has two movements: rotation and translation which has approximately the same duration, that is to say about four weeks. This is the reason that from the Earths we can see always the same half lunar superfice.

We will see this situation with a simple model. We begin by placing the volunteer who plays the role of Earth and only one "Moon" volunteer with a white mask. We place the "Moon" volunteer in front of Earth, looking to the Earth, before starting to move. So if the Moon moves 90 degrees in its orbit around the Earth, it also must turn 90 degrees on itself and therefore will continue looking in front of the Earth, and so on. We will ask to the Earth volunteer if he/she can see the same face of the Moon or can see a differnet part. We repeat the same situation four times, always moving 90º. It is evident that each 90º, that is to say each week, the Earth can see always the same part of the moon, the back of the head of the voluteer is never visible.

Moon Phases model

To explain the phases of the Moon it is best to use a model with a flashlight or with a projector (which will represent the Sun) and a minimum of five volunteers. One of them will be located in the center representing the Earth and the others will situate themselves around "the Earth" at equal distances to simulate different phases of the Moon. To make it more attractive it is a good idea for each "Moon" to wear a white mask that mimics the color of the moon. They should all face the "Earth" because we know that always the Moon offers the same side to the Earth (figure 2). We will place the flashlight above and behind one of these volunteers, and begin to visualize the phases (as seen from the Earth, that is in the center). It is very easy to discover that sometimes the mask is completely light, sometimes only a quarter and sometimes not at all (because the flashlight "Sun" is behind that "Moon" and its light dazzles the scene).The greater the number of volunteer "Moons", the more phases can be seen.

Fig. 2: Earth-Moon model with volunteers (to explain the phases and the visible face of the Moon).

Earth-Moon Model

It is not so easy to clearly understand the geometry underlying the phases of the moon, and solar and lunar eclipses. For that reason, a simple model is proposed in order to facilitate the understanding of all of these processes.

Insert two nails (about 3 or 4 cm) into a 125 cm. piece of wood. The nails should be separated by 120 cm. Two balls whose diameters are 4 and 1 cm should be placed on them (figure 3).

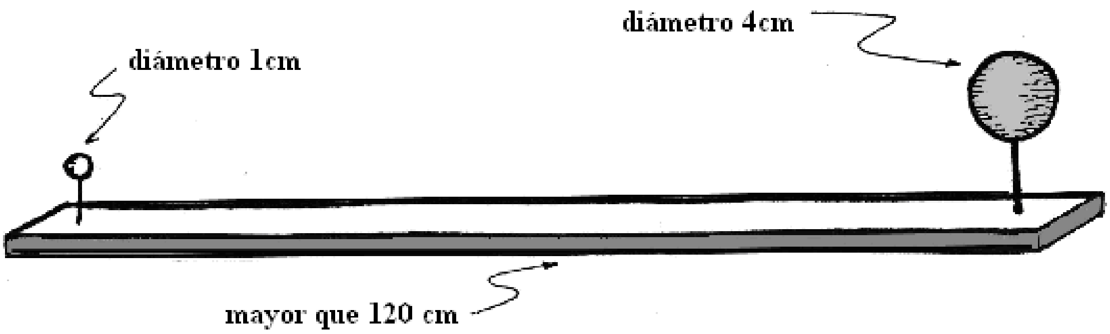


Fig. 3: Earth and Moon model.

It is important to maintain these relative sizes as they represent a scale model of the Earth-Moon system.

Earth diameter	12800 km.	→	4 cm.
Moon diameter	3500 km.	→	1 cm.
Earth-Moon distance	384000 km.	→	120 cm.
Sun diameter	1400000 km.	→	440 cm. = 4.4 m.
Earth-Sun distance	150000000 km.	→	4700 cm. = 0.47 Km.

Table 1: Distances and diameters of the Earth-Moon-Sun system.

Reproduction of Moon phases:

In a sunny place, when the Moon is visible during the day, point the model towards the Moon guiding the small ball towards it (figure 4). The observer should stay behind the ball representing the Earth. The ball that represents the Moon will seem to be as big as the real Moon and the phase is also the same. By changing the orientation of the model the different phases of the Moon can be reproduced as the illumination received from the Sun varies. The Moon-ball has to be moved in order to achieve all of the phases.

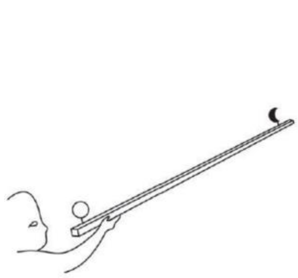


Fig.4: Using the model in the patio of the school.

It is better to do this activity outdoors, but, if it’s cloudy, it can also be done indoors with the aid of a projector as a light source.

Reproduction of Lunar eclipses

The model is held so that the small ball of the Earth is facing the Sun (it is better to use a projector to or a flashlight avoid looking at the Sun) and the shadow of the Earth covers the Moon (figure 5a and 5b) as it is larger than the Moon. This is an easy way of reproducing a lunar eclipse.

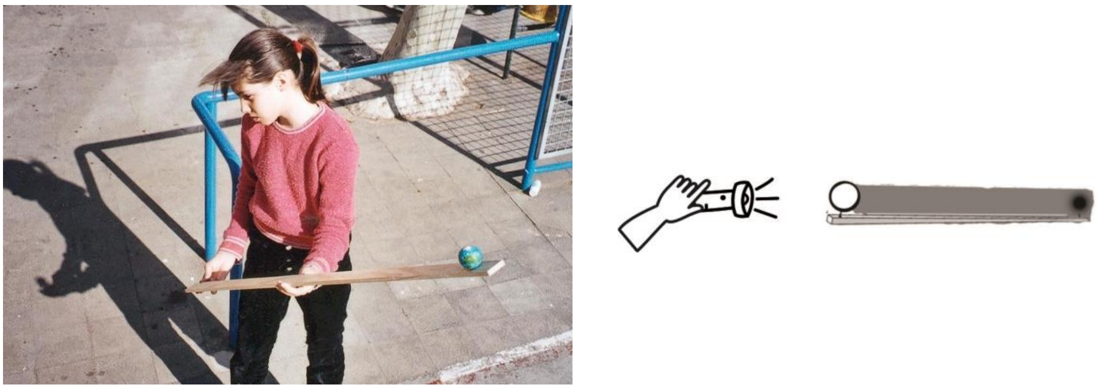


Fig.5a and 5b: Lunar eclipse simulation.

Fig. 6: Photographic composition of a lunar eclipse. Our satellite crosses the shadow cone produced by the Earth.

Reproducing the eclipses of the Sun

The model is placed so that the ball of the Moon faces the Sun (it is better to use the projector or the flashlight) and the shadow of the Moon has to be projected on the small Earth ball. By doing this, a solar eclipse will be reproduced and a small spot will appear over a region of the Earth (figures 7a, 7b and 8).

Fig. 7a and 7b Solar eclipse simulation

It is not easy to produce this situation because the inclination of the model has to be finely adjusted (that is the reason why there are fewer solar than lunar eclipses).

Fig.8: Detail of the previous figure 7a.

Fig. 9: Photograph taken from the spatial station MIR of the solar eclipse in 1999 over a region of the Earth's surface.

Observations

- A lunar eclipse can only take place when it is full Moon and a solar eclipse when it is new Moon.
- A solar eclipse can only be seen on a small region of the Earth's surface.
- It is rare that the Earth and the Moon are aligned precisely enough to produce an eclipse, and so it does not occur every new or full Moon.

Model Sun-Moon

In order to visualize the Sun-Earth-Moon system with special emphasis on distances, we will consider a new model taking into account the terrestrial point of view of the Sun and the Moon. In this case we will invite the students to draw and paint a big Sun of 220 cm diameter (more than 2 meters diameter) on a sheet and we will show them that they can cover this with a small Moon of 0.6 cm diameter (less than 1 cm diameter).

It is helpful to substitute the Moon ball for a hole in a wooden board in order to be sure about the position of the Moon and the observer.

In this model, the Sun will be fixed 235 meters away from the Moon and the observer will be at 60 cm from the Moon. The students feel very surprised that they can cover the big Sun with this small Moon. This relationship of 400 times the sizes and distances is not easy to imagine so it is good to show them with an example in order to understand the scale of distances and the real sizes in the universe. All these exercises and activities help them (and maybe us) to understand the spatial relationships between celestial bodies during a solar eclipse. This method is much better than reading a series of numbers in a book.

Earth Diameter	12 800 km	2.1 cm
Moon Diameter	3 500 km	0.6 cm
Distance Earth-Moon	384 000 km	60 cm
Sun Diameter	1400 000 km	220 cm
Distance Earth-Sun	150 000 000 km	235 m

Table 2: Distances and diameters of system Earth-Moon-Sun

Fig. 10: Sun model.

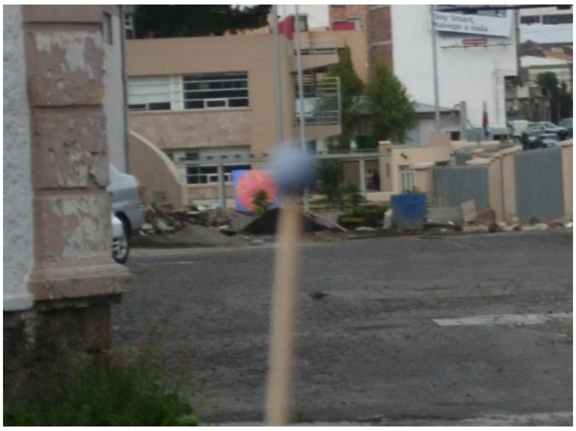


Fig. 11: Observing the Sun and the Moon in the model.

Measuring the Sun's diameter

We can measure the Sun's diameter in different ways. Here we present a simple method using a pinhole camera. We can do it with a shoebox or a cardboard tube that serves as a central axis for aluminum foil or plastic wrap.

1. We covered one end with semi-transparent vellum graph paper and the other end with a strong piece of paper or aluminum foil, where we will make a hole with a thin pin (figures 12 and 13).
2. We must point the end with the small hole towards the Sun and look towards the other end which is covered by the graph paper. We measure the diameter, d, of the image of the Sun on this graph paper.

Fig. 12 and 13: Model of the pinhole camera.

To calculate the diameter of the Sun, just consider figure 14, where we show two similar triangles.

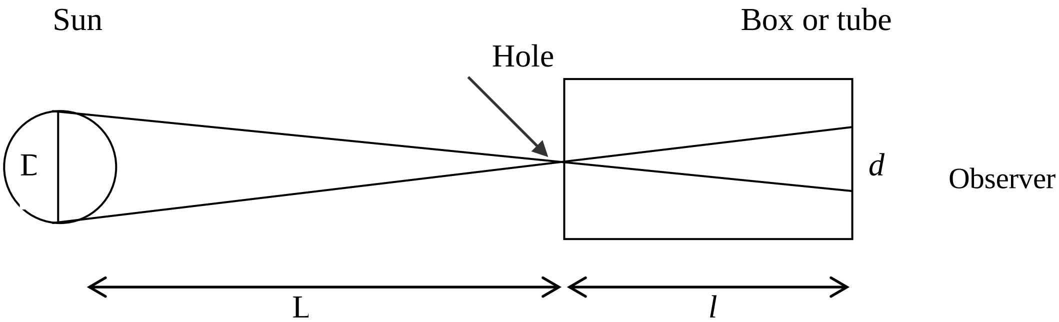


Fig. 14: Underlying geometry of calculation.

Here we can establish the relationship:

$$\frac{D}{L} = \frac{d}{l}$$

And can solve for the diameter of the Sun, D:

$$D = \frac{d \cdot L}{l}$$

Knowing the distance from the Sun to the Earth L = 150,000,000 km the tube's length l and the diameter d of the Sun's image over the screen of the graph semi-transparent paper, we can calculate the diameter D of the Sun. (Remember that the solar diameter is 1,392,000 km.).
We can repeat the exercise with the Full Moon knowing that it is 400,000 km away from the Earth.

Sizes and Distances in the Earth-Moon-Sun system

Aristarchus (310 to 230 BC) deduced the proportion between the distances and radii of the Earth-Moon-Sun system. He calculated the radius of the Sun and Moon, the distance from the Earth to the Sun and the distance from the Earth to the Moon in relation to the radius of the Earth. Some years afterwards, Eratosthenes (280-192 BC) determined the radius of our planet and it was possible to calculate all the distances and radii of the Earth-Moon-Sun system.

The proposal of this activity is to repeat both experiments as a student activity. The idea is to repeat the mathematical process and, as closely as possible, the observations designed by Aristarchus and Eratosthenes.

Aristarchus's experiment again

Relationship between the Earth-Moon and Earth-Sun distances

Aristarchus determined that the angle between the Moon-Sun line and the Earth-Sun line when the moon is in quarter phase is α=87 º (figure 15).

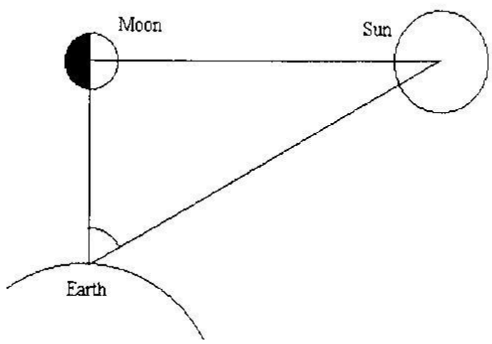


Fig. 15: Relative position of the Moon in quarter phase.

Nowadays we know that he was slightly wrong, possibly because it was very difficult to determine the precise timing of the quarter moon. In fact this angle is α = 89 º 51 ', but the process used by Aristarchus is perfectly correct. In figure 15, if we use the definition of secant, we can deduce that

$$\cos \alpha = ES/EM$$

where ES is the distance from the Earth to the Sun, and EM is the distance from the Earth to the moon. Then approximately,

$$ES = 400\ EM$$

(although Aristarchus deduced ES = 19 EM).

Relationship between the radius of the Moon and the Sun

The relationship between the diameter of the Moon and the Sun should be similar to the formula previously obtained, because from the Earth we observe both diameters as 0.5 º. So both ratios verify

$$R_S = 400 \; R_M$$

Relationship between the distance from the Earth to the Moon and the lunar radius or between the distance from the Earth to the Sun and the solar radius

Aristarchus supposes the orbit of the moon as a circle around the Earth. Since the observed diameter of the Moon is 0.5 degrees, the circular path (360°) of the Moon around the Earth would be 720 times the diameter. The length of this path is 2π times the Earth-Moon distance, i.e. $2 R_M \; 720 = 2 \pi$ EM. Solving, we find

$$EM = (720 R_M)/\pi$$

Using similar reasoning, we find

$$ES = (720 R_S)/\pi$$

This relationship is between the distances to the Earth, the lunar radius, the solar radius and the terrestrial radius.

Relationship between the distances from the Earth to the Sun and to the Moon, the lunar radius, the solar radius and the terrestrial radius.

During a lunar eclipse, Aristarchus observed that the time required for the Moon to cross the Earth's shadow cone was twice the time required for the Moon's surface to be covered (figures 16a and 16b). Therefore, he concluded that the shadow of the Earth's diameter was twice the diameter of the Moon, that is, the ratio of both diameters or radius was 2:1. Today, it is known that this value is 2.6:1.

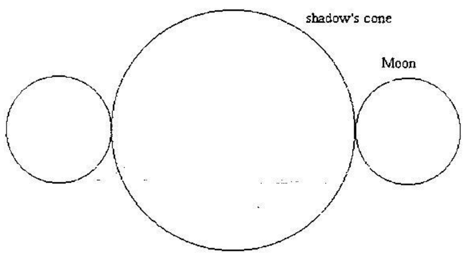


Fig. 16a: Measuring the cone of shadow.

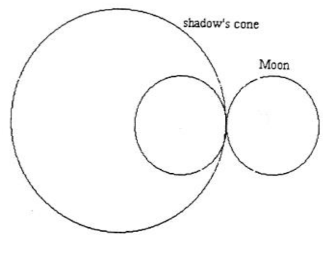


Fig.16b: Measuring the diameter of the Moon.

Final Summary

Taken into accon the last results, (figure 17)

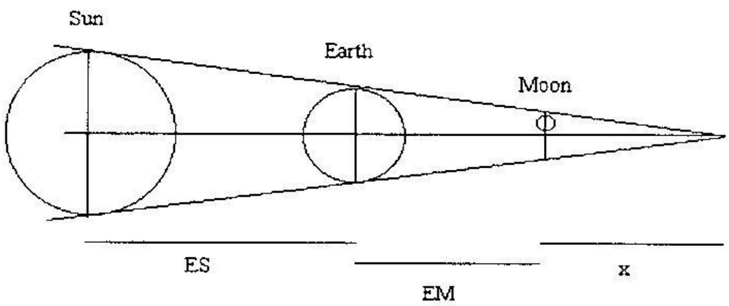


Fig. 17: Shadow cone and relative positions of the Earth-Moon-Sun system

we deduce the following relationship:

$$x /(2.6\ R_M) = (x+EM) / R_E = (x+EM+ES) / R_S$$

where x is an extra variable. Introducing into this expresion the relationships ES = 400 EM and $R_S = 400\ R_M$, we can delete x and after simplifying we obtain,

$$R_M = (401/1440)\ R_E$$

This allows us to express all the sizes mentioned previously as a function of the Earth's radius, so

$$R_S= (2005 /18)\ R_E,\ \ ES = (80200 /\pi)\ R_E,\ EM = (401 /(2\pi))\ R_E$$

where we only have to substitute the radius of our planet to obtain all the distances and radii of the Earth-Moon-Sun system.

Measurements with students

It's a good idea to repeat the measurements made by Aristarchus with students. In particular, we first have to calculate the angle between the Sun and the quarter moon. To make this measurement it is only necessary to have a theodolite and know the exact timing of the quarter moon.

So we will try to verify if this angle measures α= 87º or α= 89º 51' (although this precision is very difficult to obtain).

Secondly, during a lunar eclipse, using a stopwatch, it is possible to calculate the relationship between the following times: "the first and last contact of the Moon with the Earth's shadow cone", i.e., measure the diameter of the Earth's shadow cone (figure 17a) and "the time necessary to cover the lunar surface," that is a measure of the diameter of the moon (figure 20b). Finally, it is possible to verify if the ratio between both is 2:1 or is 2.6:1 or it is different.

The most important objective of this activity is not the result obtained for each radius or distance. The most important thing is to point out to students that if they use their knowledge and intelligence, they can get interesting results with few resources. In this case, the ingenuity of Aristarchus was very important to get some idea about the size of the Earth-Moon-Sun system.

It is also a good idea to measure with the students the radius of the Earth following the process used by Eratosthenes. Although the experiment of Eratosthenes is well known, we present here a short version of it in order to complete the previous experience.

Eratosthenes' experiment, again

Eratosthenes was the director of the Alexandrian Library. In one of the texts of the library, he read that in the city of Syena (now Aswan) the day of the summer solstice, the solar noon, the Sun was reflected in the bottom of a well, or what it is the same the stick did not produce shadow. He noted that the same day, at the same time, a stick produced no shadow in Alexandria. From this, he deduced that the surface of the Earth could not be flat, but it should be a sphere (figures 18a and 18b)

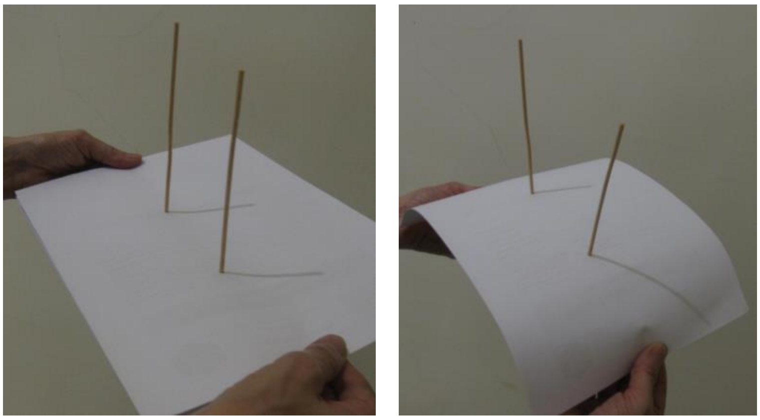


Fig. 18a an 18b: In the flat surface the two sticks produce the same shadow, but when the surface is corved sahdows are differetn.

Consider two stakes placed perpendicular to the ground, in two cities on the Earth's surface on the same meridian. The sticks should be pointing toward the center of the Earth. It is usually better to use a plumb where we mark a point of the wire to measure lengths. We should measure the length of the plumb from the ground to the mark, and the length of its shadow from the base of the plumb to the shadow of the mark.

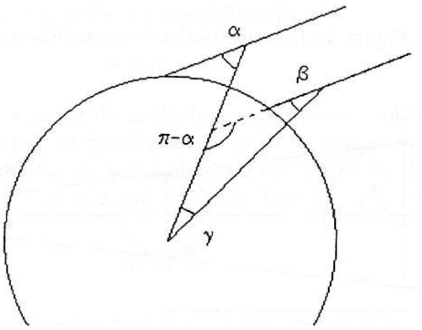


Fig. 19: Placement of plumbs and angles in the Eratosthenes experiment.

We assume that the solar rays are parallel. The solar rays produce two shadows, one for each plumb. We measure the lengths of the plumb and its shadow and using the tangent definition, we obtain the angles α and β (figure 19). The central angle γ can be calculated imposing that the sum of the three angles of the triangle is equal to π radians. Then $\pi = \pi - \alpha + \beta + \gamma$ and simplifying

$$\gamma = \alpha - \beta$$

where α and β have been obtained by the plumb and its shadow.

Finally establishing a proportionality between the angle γ, the length of its arc d (determined by the distance above the meridian between the two cities), and 2π radians of the meridian circle and its length $2\pi R_E$, we find:

$$\gamma/d = 2\pi/(2\pi R_E)$$

Then we deduce that:

$$R_E = d/\gamma$$

where γ has been obtained by the observation and d is the distance in km between both cities. We can find d from a good map. .

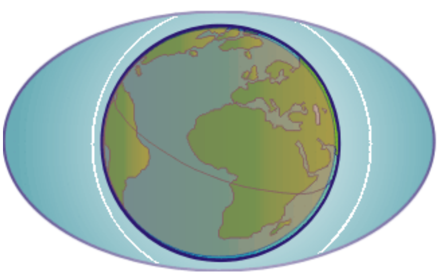
In the Eratosthenes situation, the angle β was zero and $\gamma = \alpha$, and the distance between Alejandria and Syena route, he can ontain a good result of the terrestrial radius.

It should also be mentioned that the purpose of this activity is not the accuracy of the results. Instead, we want students to discover that thinking and using all of the possibilities you can imagine can produce surprising results.

Tides

Tides are the rise and fall of sea level caused by the combined effects of Earth's rotation and gravitational forces exerted by the Moon and the Sun. The shape of the sea bottom and shore in the coastal zone also influence the tides, but to a lesser extent. Tides are produced with a period of approximately 12 ½ hours.

The tides are mainly due to the attraction between the Moon and Earth. High tides occur on the sides of the Earth facing the moon and opposite the moon (figure 120). Low tides occur in the intermediate points.



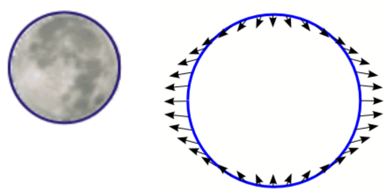


Fig. 20: Tide's effect. Fig. 21: Effect on water of the differential relative acceleration of the Earth in different areas of the ocean.

Tidal phenomena were already known in antiquity, but their explanation was only possible after the discover of Newton’s law of the Universal Gravitation (1687).

$$Fg = G\frac{m_T \times m_L}{d^2}$$

The moon exerts a gravitational force on Earth. When there is a gravitational force, there is a gravitational acceleration according to Newton's second law (F = m a). Thus, the acceleration caused by the moon on Earth is given by

$$a_g = G\frac{m_L}{d^2}$$

Where m_L is the moon mass and *d* is the distance from the moon to a point on the Earth.

The solid part of Earth is a rigid body and, therefore, we can consider all the acceleration on this solid part applied to the center of the Earth. However, water is liquid and undergoes a distinct acceleration that depends on the distance to the moon. So the acceleration of the side closest to the moon is greater than the far side. Consequently, the ocean's surface will generate an ellipsoid (figure 21).

That ellipsoid is always extended towards the Moon (figure 20) and the Earth will turn below. Thus every point on Earth will have a high tide followed by low tide twice per day. Indeed the period between tides is a little over 12 hours and the reason is that the moon rotates around the Earth with a synodic period of about 29.5 days. This means that it runs 360º in 29.5 days, so the moon will move in the sky nearly 12.2 º every day or 6.6 º every 12 hours. Since each hour the Earth itself rotates about 15 °, 6.6 ° is equivalent to about 24 minutes, so each tidal cycle is 12 hours and 24 minutes. As the time interval between high tide and low tide is about half this, the time it take for high tides to become low tides, and vice versa, will be about 6 hours 12 min.

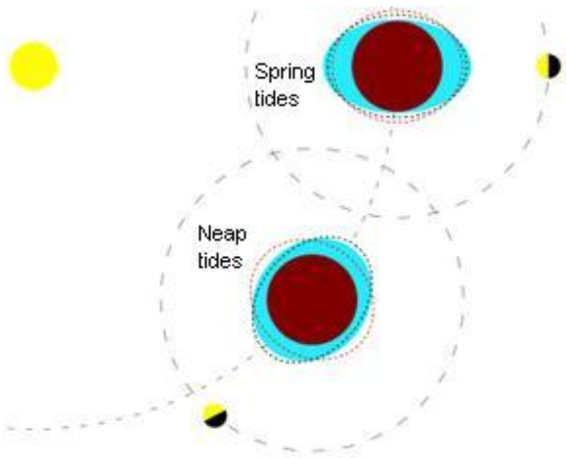


Fig. 22: Spring tides and neap tides.

Due to its proximity, the Moon has the strongest infuence on the tides. But the Sun also has influence on the tides. When the Moon and Sun are in conjunction (New Moon) or opposition

(Full Moon) spring tides occur. When the Moon and the Sun exercise prependicular gravitational attraction (First Quarter and Last Quarter), the Earth experiences neap tides (figure 22).

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